Subject: *Plane Geometry*
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Introduction to Ellipse

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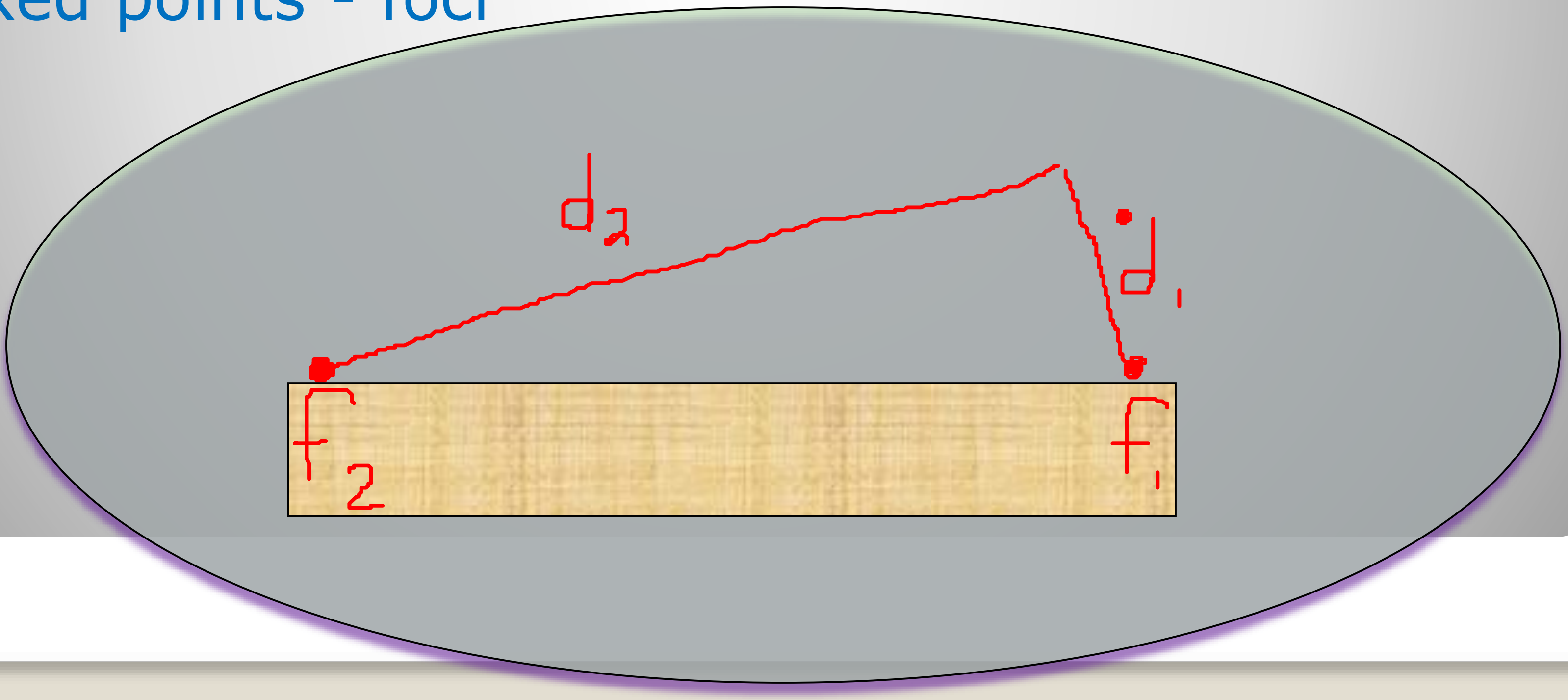
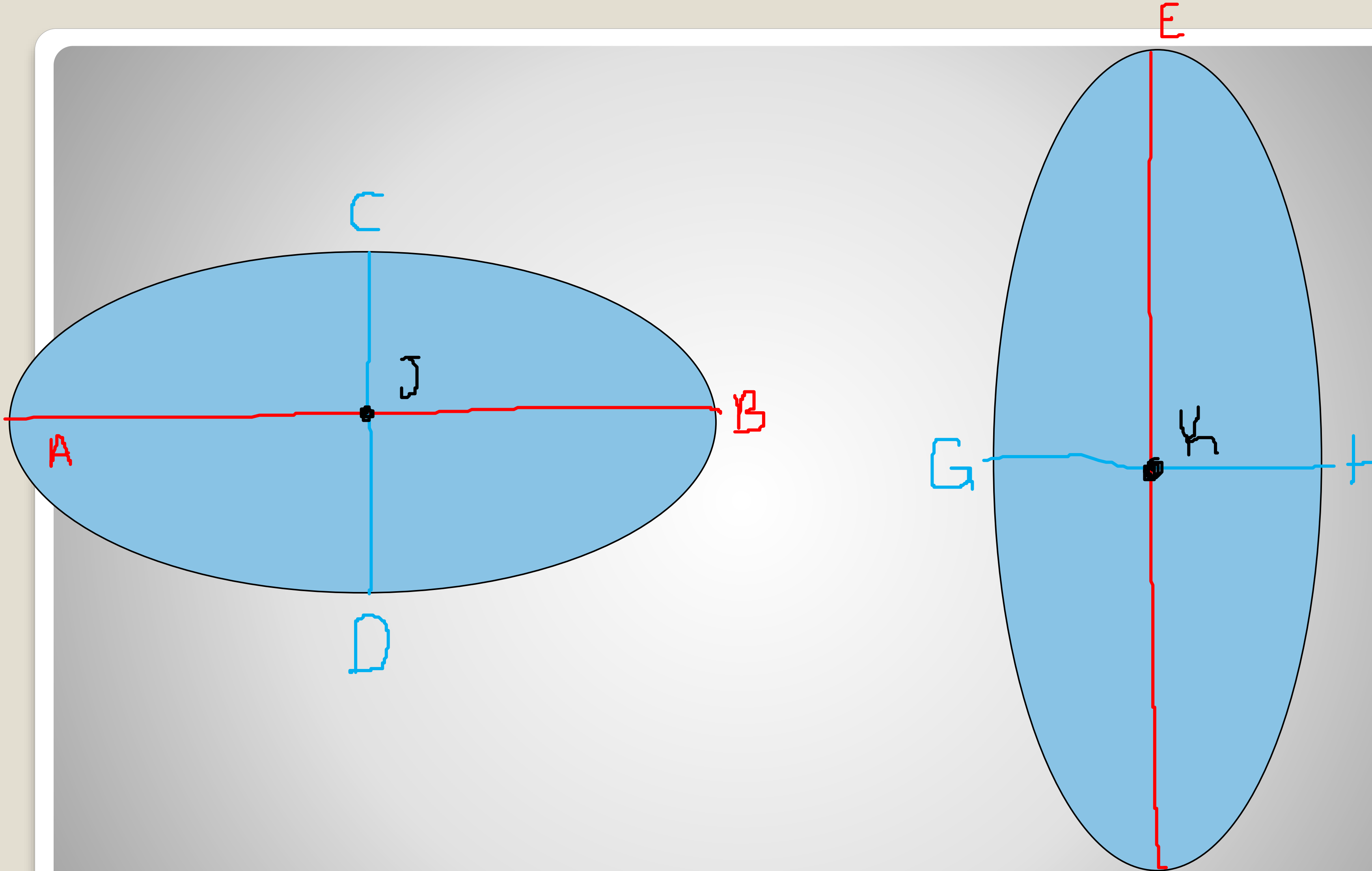
- Ellipse – An ellipse is locus of a point which moves so that its distance from a fixed point is in a constant ratio, less than one, to its distance from a fixed point.
- A set of points P in a plane such that the sum of the distances from P to 2 fixed points (F_1 and F_2) is a given constant K.
- Fixed points - foci



- Major axis – the segment that contains the foci and has its endpoints on the ellipse.
- Endpoints of major axis are vertices
- Midpoint of major axis is the center of the ellipse.
- Minor axis – perpendicular to major axis at the center
- Endpoints of minor axis are co-vertices



major

minor

Vertices: A B E F

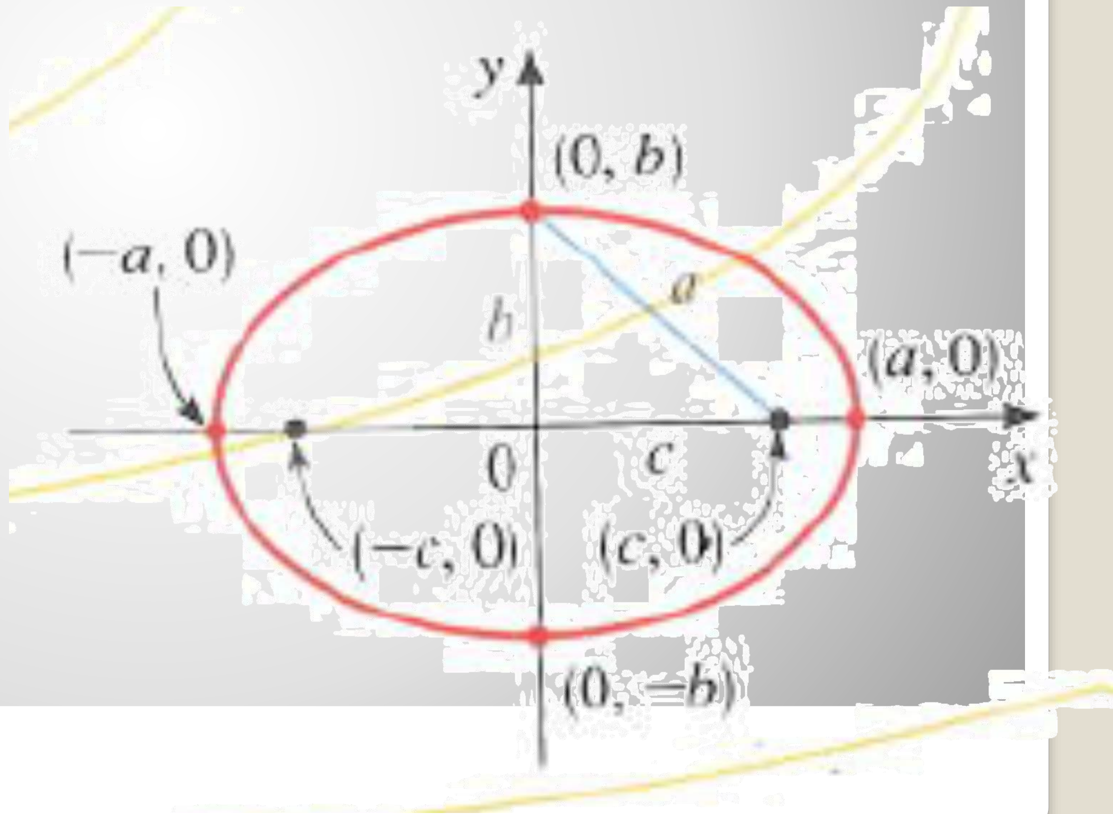
Co-Vertices: C D G H

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$		$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
x	major	y
2a	length	2a
y	minor	x
2b	length	2b
$(\pm a, 0)$	vertices	$(0, \pm a)$
$(0, \pm b)$	co-vertices	$(\pm b, 0)$
$(\pm c, 0)$	foci	$(0, \pm c)$

Length from center to foci = c

$c^2 = a^2 - b^2$

Foci are always on major axis



a > b

Write an equation if a vertex is (0, -4) and a co-vertex is (3, 0) and the center is (0, 0)

$$a = 4$$

$$b = 3$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

- Equation of a circle:
- Center (h, k) $(x-h)^2+(y-k)^2=r^2$

$$\frac{(x)^2}{a^2}+\frac{(y)^2}{b^2}=1 \qquad \frac{(x)^2}{b^2}+\frac{(y)^2}{a^2}=1$$

$(0\pm \mathrm{a}, 0)$	vertices	$(0, \pm \mathrm{a})$
$(0, 0\pm \mathrm{b})$	co-vertices	$(0\pm \mathrm{b}, 0)$
$(0\pm c, 0)$	foci	$(0, 0\pm \mathrm{c})$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \qquad\qquad \frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

(h ± a, k)	vertices	(h, k ± a)
(h, k ± b)	co-vertices	(h ± b, k)
$(h \pm c, k)$	foci	(h, k ± c)

Remember, a > b

Eccentricity of an Ellipse

- Measures how ‘circular’ the ellipse is (describes the shape of the ellipse.)

$$e = \frac{c}{a},$$ so e must be between 0 and 1 ($0<e<1$)

- .
- If *e* is close to 0 then foci are near center and more round.
- If *e* is close to 1 then foci are far from center and ellipse is elongated

Find center, foci, length of major and minor, vertices and co-vertices and graph.

$$\frac{(x-1)^2}{20}+\frac{(y+2)^2}{4}=1$$

$Center:(1,-2)$

$c=4$ $\quad Foci:c^2=a^2-b^2$

$(h\pm c,k)$ $\quad \left(1\pm 4,-2\right)$

$major:2a=4\sqrt{5}$

$minor:2b=4$

$vertices:(h\pm a,k)$

$\left(1\pm 2\sqrt{5},-2\right)$

$co-vertices:(h,k\pm b)$

$\left(1,-2\pm 2\right)$

$\left(5,-2\right)$

$\&\left(-3,-2\right)$

$\left(1,0\right)\&\left(1,-4\right)$

Find center, foci, length of major and minor, vertices and co-vertices and graph.

$x^2 + 4y^2 - 6x - 16y - 11 = 0$

$$x^2 - 6x + 4(y^2 - 4y) = 11$$

$$x^2 - 6x + 9 + 4(y^2 - 4y + 4) = 11 + 9 + 16$$

$$(x-3)^2 + 4(y-2)^2 = 36$$

$$\frac{(x-3)^2}{36} + \frac{(y-2)^2}{9} = 1$$

Find center, foci, length of major and minor, vertices and co-vertices.

$25x^2 + 4y^2 - 150x + 40y + 225 = 0$

$$25(x^2 - 6x) + 4(y^2 + 10y) = -225$$

$$25(x^2 - 6x + 9) + 4(y^2 + 10y + 25) = -225 + 225 + 100$$

$$25(x-3)^2 + 4(y+5)^2 = 100$$

$$\frac{(x-3)^2}{4} + \frac{(y+5)^2}{25} = 1$$

- Foci at (-1, 0) and (1, 0) and a = 4

$$Foci = c$$

$$c^2 = a^2 - b^2$$

$$1^2 = 4^2 - b^2$$

$$b = \sqrt{15}$$

$$\frac{x^2}{16} + \frac{y^2}{15} = 1$$

$$Center: (0,0)$$

Thanking You